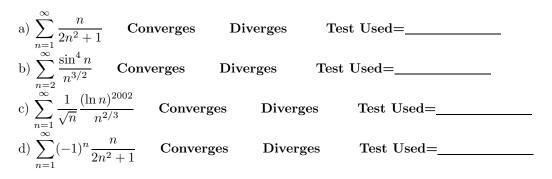# Math 143 Sample Final Exam Problems

Question 1 For each of the following sequences $\{a_n\}$, decide whether it converges or diverges and circle the appropriate word. If the sequence converges, compute the limit of the sequence and write the limit of the sequence in the blank. (Show all work.)

a) $a_n = \dfrac{3 - 4n^2 + \cos n}{\sqrt[3]{5n^6 - 4n^5 + 101}}$ **Converges** **Diverges** **Limit=** _____

b) $a_n = \sqrt{n^2 + 5n} - n$ **Converges** **Diverges** **Limit=** _____

c) $a_n = \sqrt[n]{3^{2n-3}}$ **Converges** **Diverges** **Limit=** _____

d) $a_n = \dfrac{n + (-1)^n n}{n}$ **Converges** **Diverges** **Limit=** _____

Question 2 For each of the following series decide whether the series converges or diverges and circle the appropriate word. Write the name of the test used to decide in the blank. (Show all work.)

a) $\displaystyle\sum_{n=1}^{\infty} \dfrac{n}{2n^2 + 1}$ **Converges** **Diverges** **Test Used=** _____

b) $\displaystyle\sum_{n=2}^{\infty} \dfrac{\sin^4 n}{n^{3/2}}$ **Converges** **Diverges** **Test Used=** _____

c) $\displaystyle\sum_{n=1}^{\infty} \dfrac{1}{\sqrt{n}} \dfrac{(\ln n)^{2002}}{n^{2/3}}$ **Converges** **Diverges** **Test Used=** _____

d) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \dfrac{n}{2n^2 + 1}$ **Converges** **Diverges** **Test Used=** _____

Question 3 Compute the sum of the following infinite series: $\displaystyle\sum_{n=3}^{\infty} \dfrac{(-3)^{n-2}}{2^{3n+1}}$

Question 4 Find the interval of convergence of the power series: $\displaystyle\sum_{n=0}^{\infty} \dfrac{(-1)^n (n+3)!}{n! 3^{2n}} (2x-1)^n$. Don't forget to check the endpoints!

Question 5 Use the first three non-zero terms of the power series centered at $x = 0$ for $f(x) = \dfrac{\sin(2x^3)}{x^3}$ to estimate the integral $\int_0^1 f(x)\, dx$.

Question 6 Find the first three terms of the Taylor series for $f(x) = \tan x$ centered at $x = \pi/4$.

Question 7 For the parametric curve $x = (\cos t + \sin t)$, $y = (\cos t - \sin t)$, find the equation of the tangent line at the point where $t = \pi/3$. Find the length of the curve from $t = 0$ to $t = \pi/4$. Find the area of the surface of revolution gotten by rotating the curve from $t = 0$ to $t = \pi/4$ about the y–axis.

Question 8 Find the area enclosed by the cardioid $r = 2 + 2\sin\theta$. Find the equation of the tangent line to the cardioid at the point when $\theta = \pi/6$.

Question 9 Find the length of the spiral $r = \theta$ from $\theta = 0$ to $\theta = \pi/2$.

Question 10 Find the tangent line to the curve given by

$$\mathbf{r}(t) = (3 - 1/t^2)\mathbf{i} + \sin(\pi t)\mathbf{j} - (\ln(5 - 2t))\mathbf{k}$$

at the point $(11/4, 0, 0)$.

Question 11 Find the equation of the plane containing the two (parallel) lines:

$$\mathbf{r}_1(t) = (0, 1, -2) + t(1, -2, 4) \quad \text{and} \quad \mathbf{r}_2(t) = (-5, 3, 1) + t(1, -2, 4)$$

Question 12 Find the equation of the line through the point $(3, 1, -2)$ that intersects and is perpendicular to the line given parametrically as: $x = -1 + t$, $y = -2 + t$, $z = -1 + t$.

Question 13 Let $\mathbf{u} = (a, b, 1)$, $\mathbf{v} = (1, 2, 3)$ and $\mathbf{w} = (-3, 4, 7)$. Find a value of a and b that makes $\mathbf{u}$ orthogonal to both $\mathbf{v}$ and $\mathbf{w}$.

Question 14 Find the path $\mathbf{r}(t)$ which satisfies the condition $\dfrac{d\mathbf{r}}{dt} = (t^2 - t)\mathbf{i} - (\sin t)\mathbf{j} + (16 - t^3)\mathbf{k}$ and $\mathbf{r}(0) = 3\mathbf{i} + 5\mathbf{j} - 7\mathbf{k}$

Question 15 Find the length of the curve $\mathbf{r}(t) = (\sqrt{2}t)\,\mathbf{i} + e^t\,\mathbf{j} + e^{-t}\,\mathbf{k}$, $0 \leq t \leq 2$. Find the curvature of the curve when $t = 0$.